Brownian Motion

Brownian motion:-

If some pollen grains are suspended in water and observed under a microscope, they show a continuous, random, erratic motion like a wild dance with no sign of stoppage. This type of chaotic motion is not restricted only for pollen grains and water and also it is not depend on the ~~a~~ biological origin of particles. In the field of view of the microscope, each particle is seen to spin, rise, sink and rise again. This phenomenon of chaotic motion of ~~f~~ collidal particles suspended in a liquid is known as Brownian motion.

Salient features (Brownian motion):-

(i) The motion is continuous, eternal, irregular and random.

(ii) The motion is independent of the mechanical vibration of the container.

(iii) The smaller the particle size, greater is the motion.

(iv) The lower the viscosity of the liquid, greater is the motion and conversely.

(v) The higher the temperature, greater is the motion. The ~~velocity~~ speed of two particles of same size is equal at same temperature.

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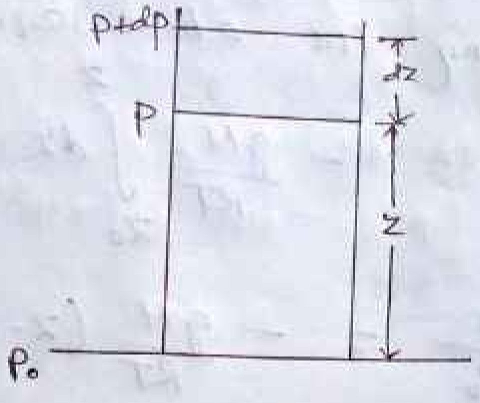
Vertical distribution of Brownian particles :-

(Law of atmosphere or Barometric equation)

Brownian particles constitute a gas which is in equilibrium under the action of gravitational force field.



Consider a column of gas is bounded by surfaces at a vertical heights z and z+dz and the pressure are p and p+dp respectively. The temperature of the gas is uniform. If ρ be the density of gas at z, then the force due to gravity on this layer by unit cross section is $\rho \cdot dz \cdot 1 \cdot g = g\rho dz$.

So, the net force on this unit cross-sectional area is $p + dp - p + g\rho dz$.

Since the layer is in equilibrium under gravity, so, $p + dp - p + g\rho dz = 0$

$$\therefore \; dp = -g\rho dz \longrightarrow (1)$$

Again, we have $pv = RT$

$$\text{or, } p \cdot \frac{M}{\rho} = RT$$

$$\therefore \rho = \frac{Mp}{RT} \longrightarrow (ii)$$

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putting the value of ρ in equation (i)
we get

$$dp = -g \cdot \frac{MP}{RT} dz$$

$$\therefore \frac{dp}{p} = -\frac{gM}{RT} dz$$

Now integrating the above equation

$$\int_{P_0}^{P} \frac{dp}{p} = -\frac{gM}{RT} \int_{z_0}^{z} dz$$

$$\text{or, } \ln \frac{P}{P_0} = -\frac{gM}{RT}(z-z_0)$$

$$\therefore \frac{P}{P_0} = e^{-\frac{Mg}{RT}(z-z_0)}$$

$$\therefore P = P_0 e^{-\frac{Mg}{RT}(z-z_0)} \longrightarrow \text{(iii)}$$

Equation (iii) is known as ~~at~~ law of atmosphere or barometric equation.

* Evaluation of Avogadro's number (N_A):-

Brownian particles obey the law of atmosphere for a ideal gas $P = nKT$ at uniform temperature $P_0 = n_0KT$. Now, putting the value of P and P_0 into eqn (iii) we get

$$n = n_0 e^{-\frac{Mg}{RT}(z-z_0)} \longrightarrow \text{(iv)}$$

where n and n_0 be the no. of particles per unit volume at height z and z_0.

Again $M = m N_A$, where m is the mass of each particles and N_A be the Avogadro's number.

Now, from eqn (iv)

$$n = n_0 e^{-\frac{N_A m g}{RT}(z - z_0)}$$

$$\frac{n}{n_0} = e^{-\frac{N_A m g}{RT}(z - z_0)}$$

or, $$\ln \frac{n}{n_0} = -\frac{N_A m g}{RT}(z - z_0)$$

or, $$\ln \frac{n_0}{n} = \frac{N_A m g}{RT}(z - z_0) \longrightarrow (v)$$

If we assuming the particles to be spherical with radius r and d, d' be the density of particle and liquid respectively.

Volume of each particles $v = \frac{4}{3}\pi r^3$.

$\therefore$ The effective mass of the suspended particle $m = \frac{4}{3}\pi r^3 (d - d')$.

Putting the value of m into eqn (v) we get,

$$\ln \frac{n_0}{n} = \frac{N_A \frac{4}{3}\pi r^3 (d - d') g}{RT}(z - z_0)$$

$$\therefore N_A = \frac{3RT}{4\pi r^3 (d - d') g (z - z_0)} \ln \frac{n_0}{n}$$

This is the equation of N_A given by Perrin.

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Problems:-

In an experiment on water suspension of gamboge at 20°C, Perrin observed an average of 49 particles/cm^2 in a layer at a certain level and 14 particles/cm^2 in another layer 60 microns higher. Find the Avogadro number, given density of gamboge = 1.1949 g/cc radius of each particle = .212 micron.

[Ans:- $N_A = 6.7 \times 10^{23}$ mol^{-1}]

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PROBLEMS

16. Calculate the mean free path and the collision frequency of hydrogen molecules at N.T.P. Given : coefficient of viscosity = 0.00008 c.g.s. unit, density of hydrogen at N.T.P. = 0.00009 g/c.c. [1.03×10^{-5} cm, 2.5×10^{10}/s] (**Burdwan Hons.**)

17. At N.T.P, the density of hydrogen is 9×10^{-5} g/c.c. and the viscosity is 8×10^{-5} in c.g.s. unit. Calculate the mean free path and the molecular diameter. If the pressure is reduced to 10^{-3} cm of mercury and if the capillary through which the gas is flowing be of radius 0.1 cm, would you expect the viscosity to change? Explain your answer and justify it by an order of magnitude calculation. Take Avogaro number, $N_A = 6.06 \times 10^{23}$, Boltzmann constant $k = 1.4 \times 10^{-16}$ erg/degree. (**Burdwan Hons.**)

18. Determine the mean free path, collision frequency and molecular diameter of air at N.T.P., given that the viscosity $\eta = 1.7 \times 10^{-5}$ Ns/m^2, mean velocity, $\bar{c} = 4.5 \times 10^2$ m/s and density $\rho = 1.29$ kg/m^3. [8.78×10^{-8}m; 5.12×10^9; 3.08×10^{-10}m]

19. Calculate the mean free path of hydrogen at standard temperature and pressure. Given, coefficient of viscosity at 273 K = 0.867 Ns/m^2 $\times 10^{-5}$, density at S.T.P. = 8.99×10^{-2} kg/m^3; density of mercury = 13.6×10^3 kg/m^3. [1.707×10^{-7}m]

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20. The coefficient of viscosity of a gas is 16.6 $\times 10^{-6}$ $Ns^{-1}m^{-1}$. Calculate the mean free path, frequency of collision and the diameter of the gas molecules. Given : $\rho = 1.25$ kg/m^3, number density $n = 2.7 \times 10^{25}$/m^3 and $\bar{c} = 450$ m/s. **(Calcutta Hons.)**

21. If the coefficient of viscosity $\eta = 1.66 \times 10^{-5}$ Ns/m^2, mean velocity $\bar{c} = 4.5 \times 10^2$ m/s, density $\rho = 1.25$ kg/m^3 and $n = 2.7 \times 10^{25}$ per cubic metre of nitrogen, calculate the mean free path, collision frequency and the diameter of nitrogen molecules.

[8.8×10^{-8} m; 5×10^9; 3.66×10^{-10}m]

22. A molecule of methane (mol. wt. =16) can be considered as a sphere having about 5 times the volume of an argon atom (at. wt. = 40). Find the ratio of viscosities and of thermal conductivities of methane and argon at N.T.P. [0.22; 0.54]

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